Overview of stochastic design strategies for wearable antennas

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***Abstract*—Variability in the fabrication process and uncertainty in deployment conditions complicate the design of wearable antennas. In this contribution, we present an overview of adapted design strategies to account for such random effects. Conventionally, overspecification of the antenna requirements accommodates for unexpected variations and, as such, ensures appropriate performance in all operational conditions. Stochastic paradigms based on polynomial chaos expansions limit these design margins to the bare essential. Their application to model fabrication tolerances, substrate compression and antenna bending is demonstrated. The specific challenges are discussed for each application scenario.**

***Index Terms*—stochastic antenna design, wearable antennas, body-centric communication.**

I. Introduction

In the past decades, there was a clear distinction between analysis and optimization paradigms for antenna design and radiowave propagation characterization. Deterministic computer-aided design tools directly based on the Maxwell equations were applied to model antenna geometries, whereas the wireless channel was described more efficiently and accurately through stochastic models. For the latter, deterministic approaches such as raytracing were found to be excessively complex and inaccurate, as it is impossible to capture all the uncertainty and variability in the propagation environment.

Novel application domains of state-of-the-art antenna systems, such as body-centric communication, the Internet of Things and the fifth-generation (5G) wireless communication system, have also introduced higher levels of variability and uncertainty in antenna performance. The use of low-cost, off-the-shelf materials and the application of cheaper production techniques result in high performance variability due to the larger tolerances on the antenna dimensions and its dielectric properties. The wide range of adverse deployment conditions encountered in these applications lead to uncertainty on the antenna performance in the actual application, altering due to bending, compression, body proximity and varying positions and orientations. Since all these effects are random, they can only be modeled accurately and efficiency through stochastic analysis and optimization techniques.

From the start, the important role of uncertainty and variability in wearable antenna design was acknowledged by many antenna designers, especially in the field of textile antenna design. To guarantee acceptable antenna performance in the presence of random performance variations, additional margins in impedance bandwidth were introduced to counter antenna detuning. Such overspecification of the antenna requirements is not economical and makes the antenna system more susceptible to interference and intermodulation distortion. To determine the additional margins on the antenna specifications, one typically analyses some extreme cases through corner analysis. Although this method works fairly well in practical wearable antenna design, a more precise analysis is required to more efficiently and accurately capture and counter variability and uncertainty in nowadays state-of-the-art applications. This requires novel stochastic paradigms that limit design margins to the bare minimum by quantifying the probability density functions of the antenna performance indicators given the statistical distributions of the design parameters and deployment uncertainties.

The conventional technique to describe uncertainty and variability given random design and deployment variables is based on a Monte Carlo analysis. In the computer-aided design process, each electromagnetic field simulation of a textile antenna requires a significant amount of CPU-time and memory. Therefore, the Mont-Carlo approach is highly time-consuming, as its accuracy only improves with the inverse of the square root of the number of realizations included in the process [1]. In this contribution, we outline a more efficient approach, which consists in replacing the full-wave simulations by a generalized polynomial chaos expansion [2], [3]. In Section II, we advocate the non-intrusive spectral projection (NISP) to construct this expansion based on a limited number of antenna simulations, which may still be performed by the conventional deterministic electromagnetic field solver. In Sections III, IV and V, we demonstrate how this stochastic design paradigm is applied to include fabrication tolerances, substrate compression and antenna bending, respectively. In each section, we outline the specific challenges pertinent to the application of the stochastic framework to each scenario.

II. Stochastic antenna design paradigm

The stochastic antenna design paradigm is schematically outlined in Fig. 1. Besides the conventional antenna design flow, seen in the left half of the figure, it includes a statistical analysis, shown in the right half of the figure, to establish the variations and uncertainty in performance of the designed antenna. First, note that the statistical procedure is non-intrusive, meaning that it does not require any modification to the conventional computer-aided design flow. Instead, it operates as an add-on to the conventional procedure.

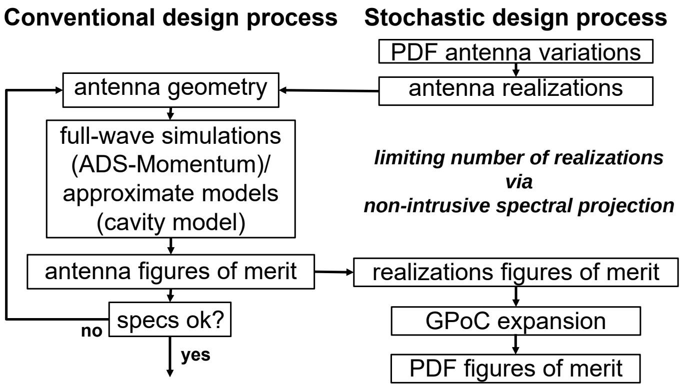

Fig. 1. Stochastic antenna design paradigm.

First, we need to establish to which variations and uncertainties the antenna is subjected. For each random variable x_k we must determine a probability density function that characterizes the variation or uncertainty of that design parameter. This already presents a first challenge, since these distributions are typically not readily available. Instead, they must be determined by experiments performed on a large set of samples. The process is complicated by the fact that multiple random variables may be correlated, requiring the construction of their joint distribution. In the following sections, we will outline for each application example how we fix the input distributions.

The next step consists of decorrelating all random variables. This is done by performing a Choleski decomposition on the covariance matrix of the design variables x_k. The resulting uncorrelated random variables u_k, each described by its cumulative distribution function $\mathcal{P}^{u_k}$ and probability density function (pdf) $d\mathcal{P}^{u_k}$, are then cast into a vector $\boldsymbol{u}$. To determine the statistical distributions of an antenna performance indicator y, we approximate the function $y = f(\boldsymbol{u})$ that relates this figure of merit to the vector of random design variables $\boldsymbol{u}$ by a generalized polynomial chaos expansion [4] of order L, given by

$$y \approx f^P(\boldsymbol{u}) = \sum_{\boldsymbol{l}=0}^{L} y_{\boldsymbol{l}}^{\boldsymbol{u}} \boldsymbol{\phi}_{\boldsymbol{l}}^{\boldsymbol{u}}(\boldsymbol{u}). \tag{1}$$

with $\boldsymbol{l} = [l_1, \ldots, l_K]$ a multi-index and with $l_1 + \ldots + l_K \leq L$. A product of orthonormal polynomials $\phi_{l,k}(x_k)$ yields an optimal expansion set, in the sense that the polynomial series converges exponentially, provided that each polynomial spans a complete orthonormal basis in Ω_k with orthonormality relation

$$\begin{aligned}\left\langle \phi_{i,k}^{u_k}(u_k), \phi_{j,k}^{u_k}(u_k) \right\rangle &= \int_{\Omega_k} \phi_{i,k}^{u_k}(u_k) \phi_{j,k}^{u_k}(u_k) d\mathcal{P}^{u_k} \\ &= \delta_{ij}.\end{aligned} \tag{2}$$

The Askey scheme provides explicit expressions for orthonormal polynomials related to well-established distributions $d\mathcal{P}^{u_k}(u_k)$. Otherwise, the orthonormal set of polynomials may be constructed by the modified Chebyshev algorithm. The orthonormality relation (2) leverages the application of the non-intrusive spectral projection (NISP) to determine the expansion coefficients $y_{\boldsymbol{l}}^{\boldsymbol{u}}$, yielding

$$\begin{aligned} y_{\boldsymbol{l}}^{\boldsymbol{u}} &= E[y(\boldsymbol{u}) \boldsymbol{\phi}_{\boldsymbol{l}}^{\boldsymbol{u}}(\boldsymbol{u})] \\ &= \int_{\Omega_1, \ldots, \Omega_L} y(\boldsymbol{u}) \phi_{l_1,1}^{u_1}(u_1) \ldots \phi_{l_K,K}^{u_K}(u_K) d\mathcal{P}^{u_1} \ldots d\mathcal{P}^{u_K}. \end{aligned} \tag{3}$$

A simple but efficient way to numerically evaluate this integral consists of applying the appropriate Gauss quadrature product rule, based on the same set of orthonormal polynomials as the ones applied in expansion (1). For a large number of random variables, tensor product rules lead to a large number of function evaluations. Stroud cubature rules may alleviate the problem, but they still suffer from the curse of dimensionality. Stochastic Testing [5], [6] avoids this problem by reducing the initial set of quadrature points to a smaller smaller set. Moreover, the expansion order in (1) may be kept small even for highly nonlinear functions $y = f(\boldsymbol{u})$ by replacing the polynomial expansion by a Padé approximation [7].

We will now demonstrate how the stochastic framework accounts for fabrications tolerances, substrate compression and bending of wearable antennas.

III. Inclusion of Geometrical Tolerances

To include geometrical tolerances in the design process, we first apply a corner analysis to determine the most sensitive design parameter(s) in a given antenna geometry. Considering, for example, the 2.45 GHz dual-polarized textile patch antenna in [8], we select the patch length L and width W as random variables. To fix their probability density functions, 100 such patches were manually cut and measured by a Nikon Veritas VM-250V microscope. The patch length L fits a Gaussian distribution with an average width $\overline{L} = 45.39$mm and standard deviation $\sigma_L = 0.13$mm, whereas its width matches a Gaussian distribution with average width $\overline{W} = 44.51$mm and standard deviation $\sigma_W = 0.16$mm. The correlation between length and width equals -0.009. Therefore, these random variables may be considered uncorrelated. For the considered distribution in patch length L, it is found that at least 75% of the textile antenna prototypes have the real part of their input impedance lying in the interval $[47.5, 52.5]\Omega$ and an imaginary part smaller than 2.5Ω, ensuring good matching with respect to 50Ω. More details can be found in [9].

IV. Modelling Substrate Compression

Next, we apply the stochastic design paradigm to study the effect of compression of textile and foam substrates on the wearable antenna's figures of merit. The problem is complicated by the fact that, during compression, a reduction in substrate height also results in a higher permittivity, due to the substrate material becoming denser. The characterization of the joint probability density function of substrate height and

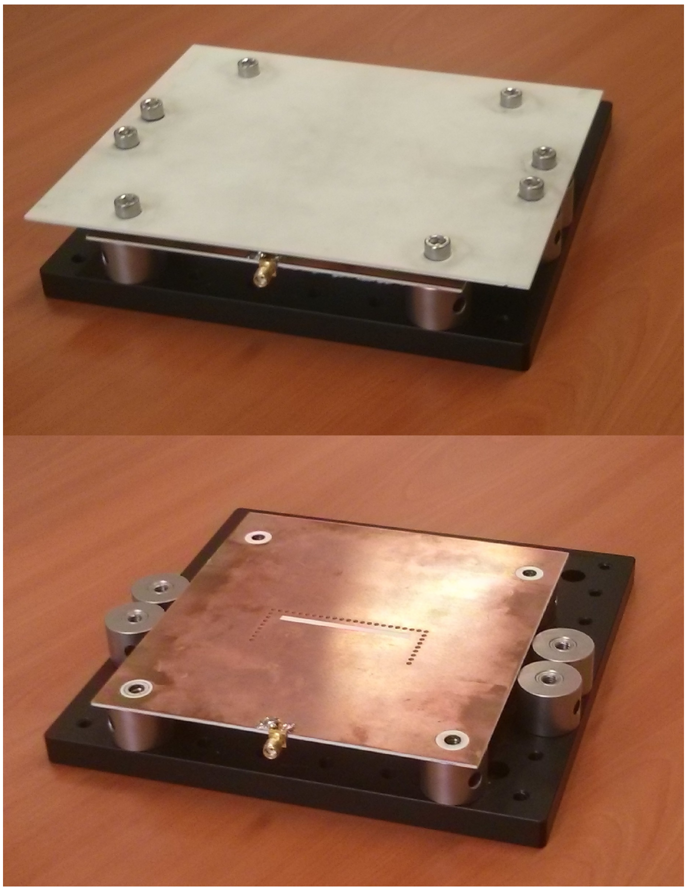

Fig. 2. Microstrip patch antenna fixture for characterization of compressed substrates.

permittivity requires a dedicated measurement fixture. With this device, shown in Fig. 2, 25 samples of a polyurethane foam with compression set 30% were compressed multiple times to 70% of their thickness. During the process, we measure the shift in resonance frequency of the aperture-coupled patch antenna for which this foam serves as a substrate. This allowed us to extract the corresponding permittivity for each sample. The joint probability density function matches a bivariate Gaussian distribution with mean height $\overline{h} = 3.35$mm and average permittivity $\overline{\epsilon}_r = 1.58$. The standard deviations of substrate height and permittivity were given by $\sigma_h = 0.34$mm and $\sigma_{\epsilon_r} = 0.05$, respectively. The correlation between both random variables was found to be $\rho = -0.68$. A Hermite-Padé approximation was applied to assess the effect of these levels of variability on the figures of merit of a probe-fed GPS textile antenna. It was found that 98% of the prototypes still satisfy the impedance matching requirement, being a return loss larger than 10dB. However, circular polarization was lost in 61% of the prototypes, since those exhibit an axial ratio larger than 3dB. More details are described in [7]

V. Describing antenna bending

We now study the uncertainty on antenna performance due to random bending of wearable antennas. This problem is highly complex, since the use of non-stretchable antenna patches leads to substrate compression while bending, depending on the radius of curvature. This process is highly complicated and impossible to describe via electromagnetic field simulations. A detailed explanation can be found in [10]. The shift in resonance frequency is modelled by a dedicated cavity model for curved textile patch antennas in which the dependency of the substrate thickness and permittivity is included via empirical formulas. To include the uncertainty due to deployment of the wearable antenna on arms with different radii of curvature, we rely on the NHANES database [11], containing the arm radius of 7056 persons larger than 1.40 m and weighing more than 40 kg. A truncated Gaussian distribution in the interval [3 cm, 8 cm], with mean radius 5.14 cm and dispersion 0.85 cm accurately fits these data. In a next step, the uncertainty on the resonance frequency of a GPS L1-band textile antenna was estimated by the stochastic design framework. It was found that 85% of the antenna prototypes have their resonance frequency in the interval [1.57, 1.58]GHz, while they resonate at 1.575GHz in planar state.

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